OPTIMIZATION OF FUZZY GUIDANCE LAW AND MEMBERSHIP FUNCTION ON THE BASIS OF GENETIC ALGORITHMS

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ABSTRACT
This article will propose a method to optimize both membership function and the system of fuzzy guidance law for self-guided missiles through genetic algorithms. With the proposed method, chromosome length will be reduced to the maximum, and only five genes will be required to encode the whole membership function of all language variables as well as the entire rule system of the fuzzy guidance law. The fuzzy guidance law will be optimized according to the adaptive function of the slip at the meeting time and the maximum acceleration of the missile. The advantage of the optimal fuzzy guidance law compared to the conventional fuzzy guidance law and the proportional navigation guidance law will be verified through simulation results.

Keywords: Fuzzy Guidance Law, Fuzzy Controller, Fuzzy Logic, Genetic Algorithm, Language Variables, Optimization, Fitness Function, Missile

INTRODUCTION
Improving management efficiency is an urgent problem in the face of increasing complexity of technological equipment, processes and systems. To design control systems for complex objects, an important role is played by solving problems of constructing adequate mathematical or simulation models and synthesizing control algorithms that provide solutions to problems under uncertainty. Intellectual decision-making systems (IDMS), imitating the principles of human thinking, are increasingly used in the management and formalization of complex objects and systems. One of the effective approaches used in the synthesis of modern IDMS is the use of the theory of fuzzy sets and fuzzy logic (Zade, 2001; Kondratenko, 2004).

Operating vague concepts in the theory of fuzzy sets maximizes the description of the control object to ordinary human speech, which greatly facilitates the construction of the model of the system. In turn, fuzzy management is implemented on the basis of rules in the form of "if that", also intuitively understandable to a person. IDMS on fuzzy logic can solve a variety of diverse management tasks in the face of uncertainty, in particular when managing mobile moving objects under extreme conditions. So, for example, in the practice of navigation, these are the tasks: a) automating the decision-making processes for the timely and adequate response of the crew when operating the vessel under extreme conditions; b) imitation modeling of the marine environment, non-stationary perturbations and corresponding decision and control algorithms for optimizing the software-algorithmic and hardware support for shipboard IDMS at the stage of their design, in particular on the basis of parallel computing systems (Zade, 2001; Kondratenko, 2004).

When creating an IDMS based on fuzzy logic, it is advisable either to develop a software module that implements the fuzzy logic inference algorithm for a specific task, or to use ready-made software developments, previously adapting them to solve the task.

So, what is a genetic algorithm? First of all, this is the method of multidimensional optimization. A method for finding the minimum of a multidimensional function. Potentially, this method can be used for global optimization, but this creates difficulties.

Genetic algorithm (GA) it is a heuristic search algorithm used to solve optimization and modeling problems by random selection, combination and variation of the desired parameters using mechanisms similar to natural selection in nature. It is a kind of evolutionary computation with the help of which optimization problems are solved using the methods of natural evolution, such as inheritance, mutations, selection and crossing-over. A distinctive feature of the genetic algorithm is the emphasis
on the use of the "crossing" operator, which performs a recombination operation of candidate solutions, whose role is analogous to the role of crossing in wildlife.

The task is formalized in such a way that its solution can be encoded as a vector ("genotype") of genes, where each gene can be a bit, a number or some other object. In classical implementations of the genetic algorithm, it is assumed that the genotype has a fixed length. However, there are variations of GA that are free of this restriction.

Some, usually random, way create a lot of genotypes of the initial population. According to Rutkovskaya D. and others (2008), they are evaluated using the "fitness function", as a result of which a certain value ("fitness") is associated with each genotype, which determines how well the phenotype described to them solves the task posed. When choosing the "fitness function" it is important to ensure that its "relief" is "smooth". From the set of solutions ("generation"), taking into account the value of "fitness", solutions are chosen (usually the best individuals are more likely to be selected), to which "genetic operators" (in most cases crossover and - mutation) are applied, solutions. For them, the fitness value is also calculated, and then the selection ("selection") of the best solutions in the next generation is made.

This set of actions is repeated iteratively, so the "evolutionary process", which lasts several life cycles (generations), is modeled, until the criterion for stopping the algorithm is fulfilled (Skobtsov, 2008). Such a criterion can be:
- finding a global, or suboptimal solution;
- exhaustion of the number of generations released for evolution;
- exhaustion of the time allowed for evolution.

According to Gladkov L.A. (2006), Genetic algorithms serve mainly to search for solutions in multidimensional search spaces. Thus, we can distinguish the following stages of the genetic algorithm:
1. Specify the objective function (fitness) for individuals of the population.
2. Create an initial population.

Genetic algorithms refer to an important direction in artificial intelligence and machine learning. Genetic programming allows you to effectively solve optimization problems that are "badly" solved by standard methods (for example, gradient methods). The main idea of genetic algorithms is that the solution is sought not by one sequence of approximate solutions, but immediately by a whole population of approximate solutions (Gladkov, Kureichik, & Kureichik, 2006).

It is also necessary to define the definition of a fuzzy controller, since in this article this concept also appears. It is regulator, built on the basis of fuzzy logic (Passino & Yurkovich, 1998; Cox, 1993). To implement a fuzzy controller, it is necessary:
1. Determine the input linguistic variables.
2. Identify the linguistic variable that we want to obtain.
3. Define the rules for the formation of the resulting variable from the input.

Used alone, to perform the functions of a linear converter with automatic control.

According to Reznik L. (1997) they are used as a part of combined optimal control systems, in which the usual regulators are used in the direct circuit, and in the additional circuit, fuzzy regulators are used that adjust the gain factors of the direct loop regulator depending on the changing conditions.

Used in solving problems of algorithmic processing of information from the studied object (filtration problems). In noise-proof adaptive systems of automatic control. In systems with a fuzzy sequential procedure for testing statistical hypotheses.

And fuzzy logic also should be defined. It is a branch of mathematics that is a generalization of classical logic and set theory, based on the notion of fuzzy set first introduced by Lutfi Zadeh in 1965 as an object with a membership function of an element to a set that takes any values in the interval 0 -
1, and not just 0 or 1. On the basis of this concept, various logical operations on fuzzy sets are introduced and a concept of a linguistic variable is formulated, with fuzzy sets as its values.

The subject of fuzzy logic is the study of reasoning under conditions of fuzziness, blurriness, similar to reasoning in the usual sense, and their application in computer systems (Kruglov & Golynov, 2000). At present, there are at least two main areas of research in the field of fuzzy logic:

- fuzzy logic in the broad sense (theory of approximate calculations);
- fuzzy logic in the narrow sense (symbolic fuzzy logic).

The basic concept of fuzzy logic in the broad sense is a fuzzy set defined by the generalized concept of a characteristic function. Then we introduce the notions of union, intersection and complementing of sets (through the characteristic function, you can specify in different ways), the notion of a fuzzy relation, and also one of the most important concepts is the concept of a linguistic variable.

Generally speaking, even such a minimal set of definitions makes it possible to use fuzzy logic in some applications, for most, you must also specify the output rule (and the implication operator).

A linguistic variable is in the theory of fuzzy sets, a variable that can take the meanings of phrases from a natural or artificial language. For example, the linguistic variable "speed" can have the values "high", "medium", "very low", etc. Phrases, the value of which the variable takes, in turn are the names of fuzzy variables and are described by a fuzzy set.

Since the proportional navigation (PN) guidance law was introduced in 1943 (Shneydor, 1998), it has become the most commonly used guidance law (Debasish, 2012; Chun-Liang & Hui, 1999; Vathsal & Sarkar, 2005; Zarchan, 1994). The success of the PN guidance law was not only due to the mathematical simplicity, ease of actualization, and the low demand for information but also due to the fact that the target at this time was large, moved slowly and had a poor maneuverability. Today, however, the aircraft are generally reduced in size, their maneuverability is improved and their moving speed is higher. Therefore, the guidance system is required to be fast and accurate.

Born in 1965, fuzzy logic has achieved strong and rapid developments and has been applied in various fields (Reznik, 1997). The success of fuzzy controllers is due to the fact that: Fuzzy controllers still ensure the quality when the working conditions vary widely; they can work under the impacts of interference or other uncertainty factors; There is no need to have an exact mathematical model of the controlled object. In the guidance field, the fuzzy logic has been applied to improve the quality of PN guidance law (Creaser, Stacey & White, 1998; Rajasekhar & Sreenatha, 2000; Trung-Dung, Tran & Duc-Vuong, 2012). However, in these studies, the authors selected the structure and parameters of the fuzzy controllers completely based on subjective experiences, or “trial and error” method. Hence, it is difficult to assert whether the received results are really optimal or not. To overcome this weak point, D.S Deshkar et al. (2011) used the genetic algorithm (GA) to optimize the membership function of fuzzy controllers and the fuzzy rule system was chosen based on subjective experiences of the authors. But to optimize membership functions, the authors would need a 35-bit binary string to encode their parameter set. This delayed the genetic algorithm implementation time and made it difficult to apply the guidance law in real time.

In order to improve the quality of the proportional navigation guidance law, this article will propose an optimal Proportional Navigation guidance law - Fuzzy Genetic Algorithms (PNFGA) that can automatically adjust the fuzzy rule system and membership functions to gain the highest guidance effectiveness. The article combines the membership function presentation method and the proposed fuzzy rule system with the real number encoding method. As a result, the chromosome length is reduced to the maximum and the genetic algorithm implementation time is reduced significantly (Yemelyanov, Kureichik, & Kureichik, 2003; Kureichik, Lebedev, & Lebedev, 2006).

The theory of fuzzy sets in a certain sense reduces to the theory of random sets and thus to the theory of probability. The basic idea is that the value of the membership function can be considered as the probability of covering the element with some random set.
However, in practical applications, the apparatus of the theory of fuzzy sets is usually used independently, acting as a competitor to the apparatus of probability theory and applied statistics (Kofman, 1982).

METHODOLOGY

Develop the Proportional Navigation Fuzzy Guidance Law

This section will develop a proportional navigation fuzzy (PNF) guidance law based on the PN guidance law. Nature of the PN guidance law is to create guidance commands to ensure that the rotational speed of the missile’s velocity vector is proportional to the rotational speed of the line of sight (LOS) (Shneydor, 1998; Debasish, 2012). The mathematical expression describing the PN guidance law is written as follows:

\[ n_C = N'V_C \dot{\lambda} \]  

(1)

in which: \( n_C \) is a normal acceleration command of the missile; \( N' \) is the guidance coefficient; \( V_C \) is the approach velocity and \( \dot{\lambda} \) the LOS rotational speed.

In fact, determining the value of the approach velocity is often difficult or impossible. While the guidance coefficient \( N' \) is usually selected empirically and receive an integer and fixed value in the whole guidance process, which reduces the effectiveness of the guidance. To overcome difficulties in determining the parameters \( V_C \)and\( N' \), the article proposed PNF guidance law with the structure as shown in Figure 1. In which: missile, the target is considered as the material point; the missile is modeled as an inertia stage with the time constants; \( T_M \); \( n_C \) and \( N' \) which are the normal acceleration commands and missile normal acceleration respectively; \( R_{TM} \) is the distance between the missile and the target; \( \lambda \) is the line of sight angle; \( \beta \) is the angle coordinated by the target velocity vector and the OX axis inverse in the positive direction. Here are descriptions of the structure of the block of PNF guidance laws.

\[ \frac{1}{s^2} \]

[Diagram of PNF Guidance Law Structure]

**Figure 1.** Diagram of PNF Guidance Law Structure

Input linguistic variables: The PNF guidance law has 2 input linguistic variables: LOS angular velocity \( \dot{\lambda} \) with 7 linguistic values \{BN, MN, SN, Z, SP, MP, BP\} corresponding to \{large, average, small, zero, positive small, positive average, positive big\} as shown in Figure 2. The second input linguistic variable is the LOS angular acceleration \( \ddot{\lambda} \) with three linguistic values \{SN, Z, SP\}.
corresponding to \{small, zero, positive small\} as shown in Figure 3. Physical values of the input variables are standardized to the segment \([-1, 1]\) through the pre-processing blocks.

![Figure 2. Presentation of Parameters Defining the Membership Function of the Linguistic Variable](image)

Output linguistic variables: Output linguistic variables of the missile normal acceleration command \(n_C\) have seven linguistic values similar to the input language variables \(\lambda\) (Figure 2). The standardized values of the output variables are converted into command values through the post-processing blocks.

**Fuzzy rule system:** The PNF guidance law uses the system of 21 fuzzy rules in the general form as shown in Table 1. In which, \(R_i, \ i = 1, 2, \ldots, 21\) can get one of the 7 linguistic values of the linguistic variables. \(n_C\).

**Table 1. Fuzzy Rule System: Optimization of Membership Functions and Legal Basis of Proportional Navigation Fuzzy Guidance Law by Genetic Algorithms**

<table>
<thead>
<tr>
<th>(n_C)</th>
<th>(\lambda)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BN</td>
<td>MN</td>
</tr>
<tr>
<td>(\ddot{\lambda})</td>
<td>SN</td>
</tr>
<tr>
<td>Z</td>
<td>R₈</td>
</tr>
<tr>
<td>SP</td>
<td>R₁₅</td>
</tr>
</tbody>
</table>

**Optimization of the Membership Function**
From Figure 2, each membership function of the two linguistic variables is defined by a set of numbers \((l_i, c_i, r_i)\) with \(i = 1..7\). However, with the constraints between the parameters shown in Figure 2, to determine 7 sets of numbers, we just need define 7 parameters \(c_i\) with \(i = 1..7\). The article proposes a method to determine \(c_i\) of the input \(\lambda\) linguistic variables and the output \(n_C\) linguistic variables according to the following expression:

\[
e_{i+4} = \text{sgn}(i) \cdot g_c \cdot |i|^q_c \quad \text{where} \quad i = \frac{n - 1}{2}, \ldots, 0, \ldots, \frac{n - 1}{2}
\]

in which:
- \(n = 7, 0 < q_c < 4\) are the quantity of membership functions and the partition coefficient of the membership function respectively?
- \(\text{sgn}(\cdot)\) is the sign function
From Figure 3, to identify three membership functions of the input linguistic variables, we just need to specify the parameter $g_c$.

Thence, to determine the shape of all 17 membership functions of all of these three linguistic variables, the article uses three genes $(q_{c1}, c_{21}, q_{c2})$ for the optimization process by GA.

**Optimization of the Fuzzy Rule System**

In order to demonstrate the complete fuzzy rule-based system while using parameters at the least, the article will propose the concept of *Fuzzy rule coordinate system*. *Fuzzy rule coordinate system* allows defining the fuzzy rule system with only two parameters.

**Method of building the fuzzy rule coordinate system:**

On each axis of the coordinate system $O\hat{\lambda}\hat{\lambda}$ we represent fuzzy sets of the input linguistic variables by points so that these points are evenly distributed on the segment $[-1, 1]$. In Figure 4, the horizontal axis is the axis $\hat{\lambda}$ representing seven fuzzy sets including: BN, MN, SN, Z, SP, MP and BP. Similarly, the vertical axis represents three fuzzy sets of linguistic variables $\hat{\lambda}$ including: SN, Z and SP.

Construct a square base going through four points with the following coordinates:

\[\{(x_1, y_1), (x_2, y_2), (1, 1), (-1, -1)\}\]

From the points representing the fuzzy sets of the input linguistic variables, we construct grid lines passing through these points so that the grid lines are perpendicular to the axis containing those points and lying in the square base. The intersection of the grid lines is called the output point.

![Figure 4. The Fuzzy Rule Coordinate System Used to Define the Fuzzy Rule System](image-url)

**Define the fuzzy rule system from the fuzzy rule coordinate system:**

**Table 2. The Fuzzy Rules System Illustrates for the Method of “Fuzzy Rule Coordinate System”**

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Thus, a fuzzy rule system will be defined by two parameters: \( q_x \) and \( \theta \).

**Optimization of Proportional Navigation Fuzzy Guidance Law by Genetic Algorithms**

**Encoding:** Using the real number encoding method, each solution (a set of typical parameter defining the membership functions and the fuzzy rule system) of the PNF guidance law optimization problem is internal\((q_{c_1}, q_{c_2}, q_{n_c}, q_x, \theta)\).

**Adaptive function:**

\[
\text{fitness} = \frac{1}{J(d_f, n_{c_{max}})} = \frac{1}{k_1d_f + k_2n_{c_{max}}}
\]

In which: \( k_1 \) and \( k_2 \) are the weights adjusting the role of two parameters: slip \( (d_f) \) maximum normal acceleration command \( (n_{c_{max}}) \) for the optimization process.

**Population size:** Population size \( N = 30 \).

**Stop conditions:** Genetic algorithms stop when they reach 100 generations or if the adaptive function changes no more than \( 10^{-6} \) after 30 generations.
Genetic operations: Selective linear ranking; hybrid $BLX - \alpha$ with $\alpha = 0.5$ and probability of hybridization $p_c = 0.8$; Inconsistent mutation with the parameter $b = 5$ and probability of mutation $p_m = 0.01$.

**FINDINGS**

To prove the effectiveness of the PNFGA guidance law achieved after the optimization by GA, in this part, the article will survey three guidance law simultaneously: The PN and PNF guidance laws with a set of parameters defining the membership functions $(q_{c^1}, c_2^1, q_{c^2}) = (1, 0, 1)$ and the fuzzy rule system is defined according to the document (Vural, 2003), and the PNFGA guidance law with the survey hypothesis as follows:

- **Missile parameters:**
  - Initial coordinates of the missile: $(R_{Mx}, R_{My}) = (0,0) m$.
  - Missile velocity: $V_M = 1000 m/s$ Guidance coefficient for PN guidance law: $N' = 3$.
  - The missile is considered to be an inertia stage with the time constant $T_M = 1 s$.

- **Target parameters:**
  - Target velocity: $V_T = 400 m/s$.
  - The initial coordinates of the target: $(R_{Tx}, R_{Ty}) = (15000,10000) m$.
  - One-sided maneuvering target according to the rules:

$$n_T = \begin{cases} 
0, & t < t_{cd} \\
nt_0 \times g, & t_{cd} \leq t \leq t_{0cd} \\
0, & t > t_{0cd} 
\end{cases}$$

with $g = 9.8 m/s^2$; $nt_0 = 7$; $t_{cd} = 2 s$ and $t_{0cd} = 10 s$.

**Simulation Results**

![Graph](image1)

**Figure 5.** The Maximum Adaptive Function Value Through Generations When Performing GA

![Graph](image2)

**Figure 6.** Membership Functions of Input Linguistic Variables
Table 3. Received Optimal Fuzzy Rule System

<table>
<thead>
<tr>
<th>$n_c$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BN</td>
</tr>
<tr>
<td>SN</td>
<td>SN</td>
</tr>
<tr>
<td>Z</td>
<td>MN</td>
</tr>
<tr>
<td>SP</td>
<td>BN</td>
</tr>
</tbody>
</table>

Table 4. Slip and Normal Acceleration at the Meeting Time of Three Guidance Laws

<table>
<thead>
<tr>
<th></th>
<th>PN</th>
<th>PNF</th>
<th>PNFGA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slip at the meeting time $d_f$ (m)</td>
<td>0.518</td>
<td>0.104</td>
<td>0.026</td>
</tr>
<tr>
<td>Maximum normal acceleration (g)</td>
<td>25.723</td>
<td>4.560</td>
<td>3.587</td>
</tr>
</tbody>
</table>

Figure 7. Membership Functions of Input Linguistic Variables

Figure 8. Membership Functions of Output Linguistic Variables

Figure 9. Missile Trajectory - Target

Figure 10. Missile Normal Acceleration Command

Figure 9 depicts the missile trajectory and targets in the implementation of the three guidance laws before the target starts to maneuver ($t < 2$ s), the trajectory of the three guidance laws has no difference. When the targets are in the maneuvering phase ($10 \leq t \leq 2$), with the membership functions and optimal linguistic variables (Figures 6, 7 and 8) and the optimized fuzzy rule system.
The PNFGA guidance law has the smallest curve when approaching the target and meeting the target at the earliest. In addition, because the PNFGA guidance law is optimized according to the slip and the maximum normal acceleration command, among three guidance laws, the PNFGA law’s maximum normal acceleration command and the slip at the meeting time are the smallest (Figure 10, Table 4).

Figure 5 shows the adaptive function value when performing GA. It can be found that the GA implementation process is fairly efficient because the initial adaptive function value increases rather rapidly and gradually stable afterwards. This demonstrates that the genetic algorithms with the proposed encoding method works well, GA is converging into the general optimal solution.

CONCLUSION

According to Samoilenko V.I., Puzyrev V.A. and Grubin I.V. (1994), optimal control is the task of designing a system that provides for a given control object or process a control law or a control sequence of actions that provide a maximum or a minimum of a given set of system quality criteria.

The task of optimal control includes the calculation of the optimal control program and the synthesis of the optimal control system. Optimal control programs, as a rule, are calculated by numerical methods for finding the extremum of a functional or solving a boundary value problem for a system of differential equations (Moiseev, 1975). The synthesis of optimal control systems from the mathematical point of view is a problem of non-linear programming in function spaces (Moiseev, 1975). To solve the problem of determining the optimal control program, a mathematical model of a controlled object or process is constructed that describes its behavior over time under the influence of control actions and its own current state (Tabak & Kuo, 1975). To solve optimal control problems with incomplete source information and in the presence of measurement errors, the maximum likelihood method is used (Moiseev, 1975). An optimal control system capable of accumulating experience and improving its work on this basis is called a learning system of optimal control (Tsypnik, 1970). The actual behavior of an object or system is always different from software due to inaccuracy in the initial conditions, incomplete information about external disturbances acting on the object, inaccurate implementation of program control, etc. Therefore, to minimize the deviation of the behavior of the object from the optimal, an automatic control system is usually used (Alexandrov, 1989). With the method of representing parameters determining the membership functions through the expression (2) and demonstrating the fuzzy rule system with the fuzzy coordinate system (Figure 4) and the expression (3), the article has significantly reduced the chromosome length when performing GA compared to other authors' studies. This allows optimizing both the membership functions and the fuzzy rule system of the fuzzy guidance law simultaneously without affecting the GA implementation time too much. The quality of the optimal fuzzy guidance law has been verified through the simulation results.

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