ON VERIFICATION OF FORMAL MODELS OF COMPLEX SYSTEMS

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ABSTRACT

The paper considers the application of the theory of Diophantine equations for the analysis of formal models of complex systems, in particular, Petri component models. For the component-place, the corresponding to this net SLIDE has been examined, its consistency has been established, which means that the network always has the ability to go from one given state to another. After analyzing the work of the *CN*-net, taking into account the work of the component-place, we come to the conclusion that all the listed properties of the constructed model are preserved.

Keywords: verification, formal model, complex system

INTRODUCTION

Investigation of processes and systems of any complexity is effectively carried out by constructing and studying the properties of their models. To implement this approach, it is necessary to build a qualitative model of the system under study. Petri component model (CN-net) can be used as this model [1, 2]. Petri component model is an optimal extension of Petri nets (PN) for constructing models of systems with parallelism, characterized by a large number of interacting processes and their considerable dimensionality. The construction of the CN-net allows us to go from the initial detailed model to the simplified description, which guarantees the reliability of the conclusions about the properties of the detailed model of the system under investigation [3].

Subjecting the resulting product to formal analysis using, for example, linear algebra methods [4], we can obtain important information about the structure and behavior of the system being simulated. Many properties of the PN are studied by methods of linear algebra, both static and dynamic ones. In this case, when verifying Petri component models, it is necessary to solve both systems of linear homogeneous Diophantine equations (SLHDE), and systems of linear inhomogeneous Diophantine equations (SLIDE).

The possibilities of verification of CN-models of complex systems are considered in the work on the model of the railway station with the use of SLHDE for finding of S - and T - invariants of a given model as well as SLIDE – for establishing the possibility of functioning of the subnet (component-place) of the *CN*-model.

PETRI MODEL ANALYSIS TOOLS

For PN, $N = (S, W, M_0)$ (where $S = (P, T, F)$ is net, $W : F \rightarrow N$ – function of the multiplicity of the arcs, $F \subseteq P \times T \bigcup T \times P$ – the incidence relation, M_0 – initial markup of the net) with m places and n transitions, a matrix of incidence A is constructed, sized $m \times n$ with integer coefficients $a_{ij} = I(t_j, p_i) - I(p_i, t_j)$, where I is the incidence function of this PN, and the coefficients a_{ij} represent the number of tokens that move, change and are added to the location p_i when the transition t_j occurs in the net N. According to the incidence matrix A, a system of equations of states of a given PN is constructed

$$Ax = d, \qquad\qquad (1)$$

where $d = M_k - M_0$ (M_0, M_k – respectively, the initial and final grid layouts of net N),

$x = \sum\limits_{j=1}^{k} u_j$, where u_k – control vector consisting of $n-1$ zero components and one j component

equal to 1, signaling the operation at the k step of the transition t_j. If the markup M_k is reachable from the markup M_0, the system (1) always has a solution [5].

The solution x of SLHDE $Ax = 0$, when in (1) $M_k = M_0$, is called T-invariant of the net N. The solution y SLHDE $A^T y = 0$ is called S-invariant of the net N.

The calculation of S- и T-invariants for a given PN makes it possible to verify such properties as reachability, boundedness, repeatability, conservatism, consistency. A special role in solving the problems of boundedness is played by minimal generating sets of S- and T-invariants [6]. The generating set of S-invariants (T-invariants) is called the minimal generating set of S-invariants (T-invariants), if there is no non-empty subset of it, which is also a generating set.

It follows from the theory of Diophantine equations [5, 6, 7], that the truncated set of solutions of SLHDE, corresponding to a given PN, is the minimal generating set of S (T)-invariants, to construct this set one can use the TSS-algorithm [8].

To establish the compatibility of the SLIDE, the compatibility criterion of the SLIDE is used [7].

Theorem 1. A system S of linear inhomogeneous Diophantine equations is compatible if and only if the auxiliary system S^* is compatible.

For SLIDE

$$S = \begin{cases} L_1(x) = a_{11}x_1 + a_{12}x_2 + ... + a_{1q}x_q = b_1; \\ L_2(x) = a_{21}x_1 + a_{22}x_2 + ... + a_{2q}x_q = b_2; \\ .... \\ L_p(x) = a_{p1}x_1 + a_{p2}x_2 + ... + a_{pq}x_q = b_p \end{cases}$$

auxiliary system S^* has the form

$$S^* = \begin{cases} L_1^{'}(x) = a^{'}_{11}x_1 + a^{'}_{12}x_2 + ... + a^{'}_{1q}x_q = 0; \\ L_2^{'}(x) = a^{'}_{21}x_1 + a^{'}_{22}x_2 + ... + a^{'}_{2q}x_q = 0; \\ .... \\ L_{p-1}^{'}(x) = a^{'}_{p-11}x_1 + ... + a^{'}_{p-1q}x_q = 0; \\ L_p(x) = a_{p1}x_1 + a_{p2}x_2 + ... + a_{pq}x_q = b_p, \end{cases}$$

where $b_p > 0$, and the first $p - 1$ equations are obtained as linear combinations:

for all $i = 1 \; дo \; p - 1$, if $b_i > 0$, then $L_i^{'} = b_i L_p(x) - b_p L_i(x)$, if $b_i < 0$, then $L_i^{'} = -b_i L_p(x) + b_p L_i(x)$.

Theorem 2. A system S^* is compatible if and only if equation

$$d_1 u_1 + d_2 u_2 + ... + d_k u_k - t b_p = 0$$

has at least one solution $(\alpha_1, \alpha_2, ..., \alpha_k, \gamma)$ in a set N such that the coordinates of the vector $v = \alpha_1 e_1^{'} + \alpha_2 e_2^{'} + ... + \alpha_k e_k^{'}$ are a multiple of the number γ. Here $d_1, d_2, ..., d_k$ are the values $L_p(x)$ on the vectors $e_1^{'}, e_2^{'}, ..., e_k^{'}$, that are vectors of the subsystem TSS of the first $p - 1$ equations of the system S^*, $t \neq 0$, $\gamma \neq 0$, α_i, t, $\gamma \in N$, $i = 1 \; дo \; k$.

Let's consider a system S, in which the equation with a free term $b_p = 1$ can be chosen as the equation $L_p(x)$. If for the first $p - 1$ equations of the system S^* it is possible to construct TSS, then by Theorem 2 it is necessary to find at least one solution of equation

$$d_1 u_1 + d_2 u_2 + ... + d_k u_k - t = 0$$

to establish the compatibility of the system S^*, satisfying the hypothesis of the theorem.

Let any two coefficients d_i and d_j of the last equation be mutually prime numbers of opposite signs, then assigning to all u_r ($r = 1, 2, ... k$, $r \neq i, j$) and a t value 1, and using the Bezout relation for d_i and d_j and the extended Euclidean algorithm, we find u_i and u_j. Thus, the desired solution of the last equation will be constructed.

The following condition is obtained for establishing the compatibility of the SLIDE.

Theorem 3. If there is an equation $L_p(x)$ with a free term equal to 1 in the SLIDE, and if at least two values $L_p(x)$ on the vectors TSS of the first $p - 1$ equations of the system S^* are mutually prime numbers with opposite signs, then such a system is compatible.

ANALYSIS OF THE PETRY MODEL

Let's consider a Petri net from [9], shown in Fig. 1, a). This net simulates the movement of trains through some dead-end railway station, with one input track and three internal tracks. In Fig. 1, b) the model is presented as a *CN*-net. In this *CN*-net, the place P_2^* is a component-place, which is separately shown in Fig. 1, c).

Having written the incidence matrix for the *CN*-net and established, that for this net $M_k = M_0 = (1,0,1)$, we build the SLHDE $Ax = 0$. Its positive integer solutions are T-invariants of *CN*-net. The problem of the compatibility of the resultant SLHDE is solved by constructing a truncated set of solutions by an TSS-algorithm. The result is a vector $e = (1,1,1) - T$-invariant. To

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search for the S - invariants we construct a SLHDE $A^T y = 0$, the compatibility of which is also established by the TSS -algorithm. The result are vectors $e_1 = (0,0,1,)$, $e_2 = (1,1,0)$.
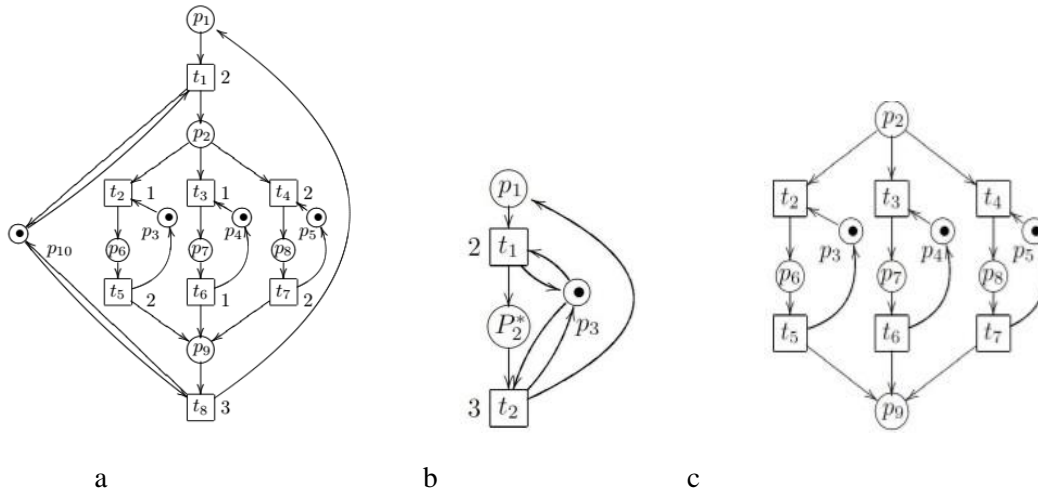

Fig. 1. Models: a) a dead-end railway station in the form of PN; b) a dead-end railway station in the form of a CN-net; c) component-place $P_2{}^{*}$

For the net that displays the component-place $P_2{}^{*}$, the initial markup $M_0 = (1,1,1,1,0,0,0,0)$ and the markup at the k - step $M_k = (0,1,1,1,0,0,0,1)$ do not coincide, so we build the SLIDE S и and its corresponding auxiliary system S^{*}.

$$S = \begin{cases} -x_1 - x_2 - x_3 = -1; \\ -x_1 + x_4 = 0; \\ -x_2 - x_5 = 0; \\ -x_3 + x_6 = 0; \\ x_1 - x_4 = 0; \\ x_2 - x_5 = 0 \\ x_3 - x_6 = 0; \\ x_4 + x_5 + x_6 = 1 \end{cases} \qquad S^{*} = \begin{cases} -x_1 - x_2 - x_3 + x_4 + x_5 + x_6 = 0; \\ -x_1 + x_4 = 0; \\ -x_2 - x_5 = 0; \\ -x_3 + x_6 = 0; \\ x_1 - x_4 = 0; \\ x_2 - x_5 = 0 \\ x_3 - x_6 = 0; \\ x_4 + x_5 + x_6 = 1 \end{cases}$$

Vectors $e`_1 = (1,0,0,1,0,0)$, $e`_2 = (0,0,1,0,0,1)$ are TSS subsystems, comprised of the first 7 equations. Values $L_8(x)$ on vectors TSS: $L_8(e`_1) = 1$, $L_8(e`_2) = 1$. Theorem 3 is not applicable. According to Theorem 2, we obtain the equation $1u_1 + 1u_2 - 1t = 0$. Let's take one of its roots, for example, $(1,0,1)$. Then, the vector-solution of the system S^{*} has the form $1e`_1 + 0e`_2 = 1(1,0,0,1,0,0)$, S^{*} is consistent. By Theorem 1, the system S is compatible to the corresponding system S^{*}.

CONCLUSION

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The analysis of the constructed Petri nets allows us to establish the properties of the formal model. Thus, from the set of S-invariants $e_1 = (0,0,1,)$, $e_2 = (1,1,0)$, *CN*-network of the dead-end railway station it is clear that all its places are covered by non-zero coordinates, this means that the network is limited.

You can get an upper score for tokens that fit anywhere, for example, for a place P_2^*: $M(P_2^*) \le \dfrac{M_0 e_1^T}{e_1(P_2^*)} = 1$ The same score for the number of tokens can be obtained for the rest of the *CN*-net in question. This means that the considered net is safe.

The presence of T-invariants indicates the possession of the net by the property of repeatability, which is interpreted as the constant movement of trains through the station. The net has the consistency property, because there is a sequence of transitions triggered on the net, which starts at least once, which can be interpreted for the station as the departure of trains.

For the component-place, the corresponding to this net SLIDE has been examined, its consistency has been established, which means that the network always has the ability to go from one given state to another.

After analyzing the work of the *CN*-net, taking into account the work of the component-place, we come to the conclusion that all the listed properties of the constructed model are preserved.

ACKNOWLEDGEMENTS

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